Tolerably Brilliant

Stats study rounds for Micah
Elementary Statistics, 14th Edition (Triola)

He only tolerates her — and somehow that became a study brand. Kind teasing. Solid stats.

TOC source: Pearson product page for Elementary Statistics 14e (9780137366446). Chapters 1–14 match that public TOC. Bonus Round: Final Review is a cumulative wrap-up (not a book chapter; Pearson's public TOC ends at 14). All examples original.

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Chapter 1 — Introduction to Statistics

Welcome! This chapter is the foundation for everything else. If you understand what data are, how they were collected, and what questions statistics can (and cannot) answer, the later math will feel much less scary.

1.1 Statistical and Critical Thinking

Key ideas

• Statistics is the science of collecting, organizing, analyzing, and interpreting data to answer questions about the world.

• A population is the complete set of people or things you care about. A sample is the smaller group you actually measure.

• A parameter describes a population (often Greek letters like μ, σ, p). A statistic describes a sample ($\bar{x}$, s, $\hat{p}$).

• Voluntary response samples (online polls, call-in shows) are usually biased — people who feel strongly respond more.

• Correlation is not causation. Two variables can move together without one causing the other.

Vocabulary

Data: collections of observations (numbers, labels, measurements).

Population: the entire group you want to learn about.

Census: collecting data from every member of the population.

Sample: a subset of the population that you actually study.

Parameter: a numerical measure of a population.

Statistic: a numerical measure of a sample.

Quantitative data: numbers that measure or count something.

Categorical (qualitative) data: labels or names that put items into groups.

Discrete data: counts that take finite or countable values (0, 1, 2, …).

Continuous data: measurements on a continuum (height, time, temperature).

Formulas

This section is mostly ideas, not formulas. Remember the mapping:

Population mean μ ↔ sample mean $\bar{x}$ | Population proportion p ↔ sample proportion $\hat{p}$

Worked Example 1: Parameter or statistic?

Step 1. A city wants to know the average commute time of all working adults who live there. Researchers survey 400 randomly chosen adults and get an average of 28.4 minutes.

Step 2. The population is all working adults in the city. The number 28.4 minutes comes from the sample of 400, so it is a statistic ($\bar{x}$ = 28.4).

Step 3. If somehow every working adult were measured and the true average were 29.1 minutes, that 29.1 would be a parameter (μ).

1.2 Types of Data

Key ideas

• Ask two questions about every variable: Is it a number that measures/counts, or a label? If it is a number, is it discrete or continuous?

• Levels of measurement (from weakest to strongest): nominal → ordinal → interval → ratio.

• Nominal: names only (blood type, zip code as a label). Ordinal: order matters but gaps are not equal (movie ratings 1–5).

• Interval: equal gaps, but no true zero (temperature in °C). Ratio: equal gaps and a true zero (income, distance, time).

Worked Example 2: Classify the data

Step 1. A fitness app records: (a) number of steps today, (b) favorite workout type (yoga/run/bike), (c) body temperature in °F, (d) satisfaction rating (poor/fair/good/excellent).

Step 2. (a) Quantitative, discrete, ratio. (b) Categorical, nominal. (c) Quantitative, continuous, interval (0 °F is not 'no temperature'). (d) Categorical/ordinal (ordered categories).

Step 3. On a test, write both the broad type (quantitative/categorical) and the level when asked.

TI-84 tip: You do not need a calculator here — just careful reading of what was measured.

1.3 Collecting Sample Data

Key ideas

• Simple random sample (SRS): every sample of size n from the population is equally likely.

• Other useful designs: stratified (sample within groups), cluster (randomly choose whole groups), systematic (every k-th item), convenience (easy but risky).

• Observational study: you watch and record without assigning treatments. Experiment: you impose treatments and compare responses.

• Good experiments use randomization, replication, and control (including blinding when possible) to reduce confounding.

• Sampling errors come from using a sample instead of a census. Nonsampling errors include bad wording, nonresponse, and biased measuring tools.

Worked Example 3: Study design check

Step 1. A café wants to know if a new loyalty app increases weekly visits. They invite the next 80 customers who walk in to try the app, then compare their visits to last month.

Step 2. This is convenience sampling of people already in the café, so results may not generalize. There is also no random assignment of 'app' vs 'no app.'

Step 3. A stronger plan: randomly select 200 loyalty-card holders, randomly assign half to get the app, track visits for 4 weeks, and compare the two groups.

1.4 Ethics in Statistics (download-only section in Triola 14e)

Even if your class skips the download-only ethics section, keep these habits: report methods honestly, do not cherry-pick results, protect privacy, and say what your sample can and cannot support. Misusing statistics can harm people.

Common mistakes

• Calling a sample average a 'population average' (or writing μ when you mean $\bar{x}$).

• Treating an online poll as a random sample.

• Saying 'A causes B' just because A and B are associated.

• Mixing up discrete counts with continuous measurements.

• Forgetting that ordinal labels (like 'agree/disagree') are not true interval numbers.

Practice Problems

1. A researcher interviews every 10th shopper leaving a mall starting at a random position. Name the sampling method.

2. True or false: The average height of all NBA players is a statistic.

3. Classify: the brand of phone a student owns.

4. Classify: the time (in seconds) to run 100 meters.

5. A news site asks readers to click yes/no on 'Should tuition be free?' Why is this a weak sample?

6. Match: μ, σ, p are usually _____ ; $\bar{x}$, s, $\hat{p}$ are usually _____.

7. Is 'placebo vs new drug, randomly assigned' an observational study or an experiment?

8. Give one example of a ratio-level measurement and one of a nominal measurement from campus life.

9. Why can temperature in °C be interval but not ratio?

10. A study finds that cities with more ice cream shops also have more swimming drownings. Does ice cream cause drownings? Explain briefly.

Answer Key — Chapter 1

1. Systematic sample.

2. False — that average for the entire NBA is a parameter.

3. Categorical / nominal.

4. Quantitative, continuous, ratio.

5. Voluntary response; respondents are self-selected and may feel strongly.

6. parameters; statistics.

7. Experiment (treatments are assigned).

8. Examples vary: ratio — credits completed; nominal — major name.

9. 0 °C does not mean 'no temperature,' so ratios like 'twice as hot' are not meaningful.

10. No — both rise in summer (confounding). Association ≠ causation.

Cheat Sheet — Chapter 1

One-page review. Fold this or screenshot it before the test.

Big picture

- Population → parameter; sample → statistic.
- Ask: how were the data collected? Can I generalize?

Data types

- Categorical vs quantitative; discrete vs continuous.
- Levels: nominal < ordinal < interval < ratio.

Sampling & studies

- Prefer SRS / well-designed probability samples over convenience or voluntary response.
- Observational study watches; experiment assigns treatments.

Red flags

- Voluntary polls, tiny samples, causation claims from observational data, loaded survey wording.

Chapter 2 — Exploring Data with Tables and Graphs

Before you compute fancy statistics, look at the data. Tables and graphs show shape, center, spread, and surprises (outliers).

2.1 Frequency Distributions for Organizing and Summarizing Data

Key ideas

• A frequency distribution lists classes (bins) and how many observations fall in each.

• Relative frequency = class frequency / n. Percentage = relative frequency × 100%.

• Choose about 5–20 classes. Class width ≈ (max − min) / number of classes, then round up to a convenient number.

• Lower and upper class limits define each bin; class boundaries are the edges with no gaps between bars later.

• Cumulative frequency adds frequencies from the first class up through the current class.

Frequency: count of observations in a class.

Relative frequency: proportion of the sample in a class.

Class width: difference between consecutive lower class limits.

Histogram-ready table: classes with no gaps, covering the full data range.

Relative frequency: f / n

When to use: Summarizing what share of the data sits in each class.

Rough class width: $(\max - \min) / (\#\ \text{of classes})$

When to use: Planning a frequency table; then round the width up.

Worked Example 1: Build a frequency table

Step 1. Data (minutes of study last night for 16 students): 20, 35, 40, 25, 55, 30, 45, 50, 60, 35, 40, 25, 70, 45, 30, 40. Use 5 classes.

Step 2. Range = 70 − 20 = 50. Width ≈ 50/5 = 10. Use classes 20–29, 30–39, 40–49, 50–59, 60–69, wait — 70 needs a sixth class 70–79, or widen. Better: width 11 → awkward. Use width 10 with classes 20–29 … 70–79 (6 classes is fine).

Step 3. Frequencies: 20–29: 3; 30–39: 4; 40–49: 5; 50–59: 2; 60–69: 1; 70–79: 1. Check: 3+4+5+2+1+1 = 16. Relative frequency of 40–49 is 5/16 = 0.3125 (31.25%).

2.2 Histograms

Key ideas

• A histogram graphs class intervals on the horizontal axis and frequency (or relative frequency) on the vertical axis. Bars touch.

• Look for shape: roughly symmetric, skewed right (long right tail), skewed left, uniform, or bimodal (two peaks).

• Outliers appear as bars far from the main pile. Always ask whether an outlier is a data error or a real extreme value.

• Changing class width can hide or exaggerate features — try more than one view if something looks odd.

Worked Example 2: Read a histogram's story

Step 1. Suppose a histogram of restaurant wait times (minutes) is piled on the left with a long tail to the right.

Step 2. Shape: skewed right. Most waits are shorter; a few long waits pull the mean above the median.

Step 3. For skewed-right data, the median is often a better 'typical' wait than the mean (you will formalize this in Chapter 3).

TI-84 tip: TI-84: STAT → Edit enter data in L1. Then 2nd → Y= (STAT PLOT) → On → Histogram → Zoom → 9:ZoomStat.

2.3 Graphs That Enlighten and Graphs That Deceive

Key ideas

• Good graphs: bar graphs (categorical), Pareto charts (bars sorted by frequency), pie charts (parts of a whole — use sparingly), time-series (change over time), stemplots (show every value).

• Deceptive tricks: truncated vertical axis (bars don't start at zero), pictographs that scale area instead of height, 3-D pie slices that look bigger than they are, uneven time gaps.

• When you see a dramatic graph in the news, check the scale, the sample, and what was left out.

2.4 Scatterplots, Correlation, and Regression

Key ideas

• A scatterplot shows paired quantitative data (x, y). Look for direction (positive/negative), form (linear/curved), and strength (how tightly points follow a pattern).

• Correlation describes linear association; regression finds a line that predicts y from x. Chapter 10 goes deep — here you only need the visual idea.

• A strong scatter pattern still does not prove causation.

Worked Example 3: Sketch what you'd expect

Step 1. For hours of sleep (x) vs. next-day alertness score (y) for students, what scatterplot shape is plausible?

Step 2. Likely a positive trend up to a point (more sleep → higher alertness), possibly flattening or bending for very high sleep.

Step 3. If someone claims 'correlation r = 2.1,' that is impossible — linear correlation stays between −1 and 1.

TI-84 tip: TI-84 scatter: enter x in L1, y in L2. STAT PLOT → scatter icon → ZoomStat.

Common mistakes

• Leaving gaps between histogram bars (that would be a bar graph for categories).

• Using unequal class widths without adjusting the vertical scale.

• Forgetting to check that frequencies sum to n.

• Reading a truncated-axis bar graph as if differences are huge.

• Calling any upward scatter 'proof that x causes y.'

Practice Problems

1. For data ranging from 12 to 87 with 8 classes, what is a reasonable class width?

2. A class has frequency 14 in a sample of n = 50. Find the relative frequency and percent.

3. True or false: Histogram bars should usually touch.

4. Name the shape: most data on the right, long tail on the left.
5. Why can a pie chart with 12 tiny slices be a poor display?
6. A bar graph of 'crimes this year' starts the vertical axis at 900 instead of 0. What visual lie can that create?
7. In a scatterplot, points fall tightly along a line sloping down. Describe direction and strength qualitatively.
8. Make a frequency table with width 10 starting at 0 for: 3, 7, 12, 15, 18, 22, 24, 29, 31, 35.
9. What is one advantage of a stemplot over a histogram?
10. Give an example of a time-series question on campus (what is measured over time?).

Answer Key — Chapter 2

1. Range = 75; 75/8 = 9.375 → use width 10 (round up).
2. 14/50 = 0.28 = 28%.
3. True (for equal-width quantitative classes).
4. Skewed left.
5. Hard to compare many small categories; a Pareto/bar chart is clearer.
6. Small changes look enormous because the axis is truncated.
7. Negative association, strong, roughly linear.
8. 0–9: 2; 10–19: 3; 20–29: 3; 30–39: 2.
9. You can still see every original value (and rebuild the list).
10. Example: daily number of students in the dining hall at noon for two weeks.

Cheat Sheet — Chapter 2

One-page review. Fold this or screenshot it before the test.

Tables

- Class width ≈ (max−min)/#(classes); round up.
- Relative frequency = f/n; frequencies must sum to n.

Histograms

- Bars touch; read shape, center, spread, outliers.
- Right skew: long right tail; left skew: long left tail.

Graph sense

- Check scales, zero baseline, and whether the graph matches the data type.

Scatterplots

- Direction, form, strength; correlation ≠ causation.

TI-84

- STAT PLOT + ZoomStat for histograms and scatters.

Chapter 3 — Describing, Exploring, and Comparing Data

Now we summarize a data set with numbers: a typical value (center), how spread out it is (variation), and where a score sits relative to others.

3.1 Measures of Center

Key ideas

- Mean (average) = sum of values / n. Sensitive to outliers and skew.
- Median = middle value once data are ordered. Resistant to outliers.
- Mode = most frequent value (data can be unimodal, bimodal, or have no mode).
- Midrange = (max + min)/2 — quick but not resistant and rarely preferred.
- For skewed-right data, typically mean > median. For skewed-left, mean < median.

Sample mean: $\bar{x} = (\Sigma x) / n$
When to use: Estimating a typical value from sample data; population mean uses $\mu = (\Sigma x)/N$.

Median: middle of ordered list (average the two middle values if n is even)
When to use: When outliers or skew make the mean misleading.

Worked Example 1: Mean and median

Step 1. Quiz scores: 72, 68, 75, 70, 69, 74, 71, 73.

Step 2. Ordered: 68, 69, 70, 71, 72, 73, 74, 75. n = 8 (even), so median = (71+72)/2 = 71.5.

Step 3. Mean $\bar{x}$ = (68+69+70+71+72+73+74+75)/8 = 572/8 = 71.5. Here mean = median (nearly symmetric).

TI-84 tip: TI-84: enter data in L1 → STAT → CALC → 1-Var Stats → L1.

3.2 Measures of Variation

Key ideas

- Range = max − min (easy, but uses only two points).
- Sample variance s^2 uses n−1 in the denominator. Population variance σ^2 uses N.
- Standard deviation is the square root of variance — same units as the data.
- Larger s (or σ) means more spread. Compare variation only for similar units/context.
- Coefficient of variation CV = $(s/\bar{x})\cdot 100\%$ lets you compare relative spread across different scales.

Sample standard deviation: $s = \sqrt{[\ \Sigma(x - \bar{x})^2 / (n - 1)\ ]}$
When to use: Measuring spread in a sample (the usual case in this course).

Population standard deviation: $\sigma = \sqrt{[\ \Sigma(x - \mu)^2 / N\ ]}$
When to use: Only when you truly have the whole population.

Range rule of thumb: $s \approx \text{range}/4$
When to use: Rough check or estimate when you only know max and min.

Worked Example 2: Compute s

Step 1. Data: 12, 15, 14, 18, 16, 15, 13, 17, 14, 15. n = 10.

Step 2. Mean $\bar{x}$ = 149/10 = 14.9. Squared deviations sum to $\Sigma(x-\bar{x})^2 = 28.9$.

Step 3. s^2 = 28.9/(10−1) = 3.211… and s = √3.211… ≈ 1.792. Range = 18−12 = 6; range/4 = 1.5 (same ballpark).

TI-84 tip: 1-Var Stats reports Sx (sample) and σx (population). For classwork samples, use Sx.

3.3 Measures of Relative Standing and Boxplots

Key ideas

- z-score = (x − mean) / (standard deviation). Tells how many SDs above/below the mean a value is.
- A z near 0 is typical; |z| > 2 is unusual by a common rule of thumb; |z| > 3 is often an outlier candidate.
- Percentiles and quartiles locate relative standing. Q1 (P25), Q2 (median), Q3 (P75).
- IQR = Q3 − Q1. Outlier fences: below Q1 − 1.5·IQR or above Q3 + 1.5·IQR.
- A boxplot (box-and-whisker) shows min/Q1/median/Q3/max (or whiskers to non-outlier extremes).

z-score (sample): $z = (x - \bar{x}) / s$
When to use: Comparing values from different distributions or spotting unusual scores.

IQR & fences: $IQR = Q3 - Q1$; fences at $Q1 - 1.5 \cdot IQR$ and $Q3 + 1.5 \cdot IQR$
When to use: Building boxplots and flagging outliers.

Worked Example 3: z-score and an outlier check

Step 1. Exam mean $\bar{x} = 78$, $s = 6$. Maya scored 90. $z = (90-78)/6 = 12/6 = 2.0$ (about 2 SDs above the mean — solid, a bit unusual).

Step 2. Five-number sense for data 4, 5, 6, 7, 8, 9, 20: median = 7, Q1 = 5, Q3 = 9, IQR = 4. Upper fence = 9 + 1.5·4 = 15. The 20 is an outlier.

Step 3. On a boxplot, draw a point (or asterisk) at 20 instead of stretching the whisker all the way there.

TI-84 tip: TI-84 boxplot: STAT PLOT → boxplot icon. Use the modified boxplot option to show outliers.

Common mistakes

- Dividing by n instead of n−1 when computing sample s.
- Using the mean for heavily skewed income data when the median is asked for or more appropriate.
- Forgetting to sort before finding the median or quartiles.
- Mixing up Sx and σx on the calculator.
- Saying a z-score of 0.1 is 'extreme.'

Practice Problems

1. Find mean and median of 10, 12, 12, 15, 21.
2. If an outlier 100 is added to 10, 12, 12, 15, 21, which measure of center changes more: mean or median?
3. For 2, 4, 6, 8, compute sample s (show the formula setup).
4. Scores: mean 70, s = 5. Find z for a score of 61.
5. Q1 = 20, Q3 = 32. Find IQR and the upper outlier fence.
6. True or false: standard deviation can be negative.
7. Data are strongly skewed right. Which is usually larger, mean or median?
8. Approximate s if values range from 40 to 88 using the range rule of thumb.
9. A boxplot's left whisker is much longer than the right. What skew does that suggest?
10. CV for a sample with $\bar{x} = 50$ and $s = 4$ is what percent?
11. Why do we use n−1 for sample variance?

12. List the five-number summary pieces.

Answer Key — Chapter 3

1. Mean = 14; ordered middle = 12 (median).
2. Mean jumps from 14 to ≈28.3; median only moves from 12 to 13.5. The mean changes more.
3. $\bar{x} = 5$; $\Sigma(x-\bar{x})^2 = 20$; $s = \sqrt{(20/3)} \approx 2.582$.
4. $z = (61-70)/5 = -1.8$.
5. IQR = 12; upper fence = 32 + 18 = 50.
6. False.
7. Mean.
8. range/4 = 48/4 = 12.
9. Skewed left.
10. CV = (4/50)·100% = 8%.
11. To unbias the estimate of σ^2 from sample data (degrees of freedom).
12. Min, Q1, median, Q3, max.

Cheat Sheet — Chapter 3

One-page review. Fold this or screenshot it before the test.

Center

- $\bar{x} = \Sigma x/n$; median = middle; mode = most frequent.
- Skewed right → mean pulled right of median.

Spread

- $s = \sqrt{[\Sigma(x-\bar{x})^2/(n-1)]}$; use Sx on TI-84 for samples.
- Range = max−min; rough s ≈ range/4.

Standing

- z = (x − mean)/SD; |z|>2 unusual (rule of thumb).
- IQR = Q3−Q1; outliers beyond 1.5·IQR fences.

Boxplots

- Show five-number summary; great for comparing groups.

Chapter 4 — Probability

Probability is the language of chance. Master the rules here and hypothesis tests later will feel like a sequel, not a new language. You've got this!

4.1 Basic Concepts of Probability

Key ideas

- A probability is a number from 0 to 1 (or 0% to 100%) describing how likely an event is.
- Classical approach: P(A) = (# of equally likely ways A can happen) / (# of equally likely outcomes).
- Relative frequency: P(A) ≈ (times A occurred) / (times we tried).
- Complement: P(Ā) = 1 − P(A). Often easier to compute 'at least one' via complement.

Classical probability: $P(A) = n(A) / n(S)$

When to use: When outcomes are equally likely (fair dice, cards, random draws).

Complement rule: $P(\bar{A}) = 1 - P(A)$

When to use: When 'not A' is easier, or for 'at least one' problems.

Worked Example 1: Cards and complements

Step 1. From a standard 52-card deck, P(ace) = 4/52 = 1/13 ≈ 0.0769.

Step 2. P(not ace) = 1 − 4/52 = 48/52 ≈ 0.9231.

Step 3. P(heart or ace) needs the addition rule (next section) because the ace of hearts is in both groups.

4.2 Addition Rule and Multiplication Rule

Key ideas

- Addition ('or'): P(A or B) = P(A)+P(B)−P(A and B). If mutually exclusive, the overlap is 0.
- Multiplication ('and'): P(A and B) = P(A)·P(B|A). If independent, P(A and B) = P(A)·P(B).
- Independent means knowing A happened doesn't change P(B). Disjoint (mutually exclusive) means they can't both happen — disjoint events with P>0 are dependent!

General addition: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

When to use: Any 'or' probability; drop the overlap term only if disjoint.

Independent 'and': $P(A \cap B) = P(A)P(B)$

When to use: When events do not affect each other (with replacement, separate flips, etc.).

Worked Example 2: Or and and

Step 1. P(heart or ace) = 13/52 + 4/52 − 1/52 = 16/52 ≈ 0.3077.

Step 2. Two fair coin flips: P(both heads) = (1/2)·(1/2) = 1/4 = 0.25 (independent).

Step 3. Without replacement: bag with 3 red and 5 blue. P(two reds) = (3/8)·(2/7) = 6/56 ≈ 0.1071.

4.3 Complements, Conditional Probability, and Bayes' Theorem

Key ideas

- Conditional: P(A|B) = P(A and B) / P(B) — 'probability of A given B already happened.'
- P(at least one) = 1 − P(none).

• Bayes' Theorem updates prior probabilities after new evidence (tree diagrams help a lot).

Conditional probability: $P(A|B)=P(A\cap B)/P(B)$

When to use: When you are told that B occurred and want the chance of A.

At least one: $1 - P(\text{none})$

When to use: Birthday-style or 'at least one defect' problems.

Worked Example 3: At least one + conditional

Step 1. P(at least one head in 3 fair flips) = 1 − P(TTT) = 1 − $(1/2)^3$ = 1 − 1/8 = 7/8 = 0.875.

Step 2. Clinic: 2% have a condition. Test: 95% true positive, 10% false positive. Pick a random person who tests positive — are they likely to have it?

Step 3. Tree: P(cond∩+) = 0.02·0.95 = 0.019; P(no∩+) = 0.98·0.10 = 0.098; P(+) = 0.117. P(cond|+) = 0.019/0.117 ≈ 0.162. Only about 16% — surprising but classic Bayes!

4.4 Counting

Permutations: $nPr = n!/(n-r)!$

When to use: Order matters (passwords, race places).

Combinations: $nCr = n!/(r!(n-r)!)$

When to use: Order does not matter (committees, card hands).

• Fundamental counting rule: multiply choices along a sequence of decisions.

4.5 Simulations for Hypothesis Tests

• A simulation repeats a chance process many times to see how rare a result would be if a null claim were true — a tech-friendly preview of p-values.

Common mistakes

• Adding probabilities for 'and' instead of multiplying.
• Forgetting to subtract the intersection in the addition rule.
• Treating dependent draws (without replacement) as independent.
• Using permutations when order does not matter.
• Thinking P(A|B) equals P(B|A).

Practice Problems

1. P(rolling a 5 on a fair die)?
2. P(king or queen from 52 cards)?
3. Two independent events each with P=0.3. P(both)?
4. P(at least one tail in 2 fair flips)?
5. From 5 women and 4 men, P(both selected are women) if 2 are chosen without replacement?
6. Compute 5P2 and 5C2.
7. If P(A)=0.4, P(B)=0.5, P(A∩B)=0.2, find P(A|B).
8. True/false: mutually exclusive events with nonzero probability are independent.
9. A password is 3 letters (26) with repetition allowed. How many?
10. Bag with 2 defective out of 10. P(both defective) when drawing 2 without replacement?

Answer Key — Chapter 4

1. 1/6 ≈ 0.1667.
2. 8/52 = 2/13 ≈ 0.1538.
3. 0.09.
4. 1 − 1/4 = 0.75.
5. (5/9)·(4/8)=20/72≈0.2778.
6. 5P2=20; 5C2=10.
7. 0.2/0.5=0.4.
8. False.
9. 26^3=17,576.
10. (2/10)·(1/9)=2/90≈0.0222.

Cheat Sheet — Chapter 4

One-page review. Fold this or screenshot it before the test.

Basics

- 0 ≤ P ≤ 1; P(Ā)=1−P(A).

Or / And

- Or: add, subtract overlap. And: multiply (use conditional if dependent).

Conditional

- P(A|B)=P(A∩B)/P(B).

Counting

- nPr order matters; nCr order doesn't.

Tip

- 'At least one' → complement of none.

Chapter 5 — Discrete Probability Distributions

Discrete random variables count things: successes, defects, arrivals. Learn the checklist for a valid distribution, then the binomial and Poisson stars of the chapter.

5.1 Probability Distributions

Key ideas

- A discrete probability distribution lists each value x with P(x), where every P(x) ≥ 0 and ΣP(x) = 1.
- Mean (expected value) μ = Σ[x·P(x)]. Think long-run average.
- Variance $\sigma^2 = \Sigma[(x-\mu)^2 \cdot P(x)] = \Sigma[x^2 P(x)] - \mu^2$; $\sigma = \sqrt{\sigma^2}$.

Expected value: $\mu = \Sigma[x \cdot P(x)]$
When to use: Finding the mean of a discrete random variable.

SD of discrete RV: $\sigma = \sqrt{(\Sigma[x^2 P(x)] - \mu^2)}$
When to use: Measuring spread of a probability distribution.

Worked Example 1: Is it a distribution? Find μ

Step 1. x: 0,1,2 with P: 0.2, 0.5, 0.3. All ≥0 and sum=1 → valid.

Step 2. μ = 0(0.2)+1(0.5)+2(0.3) = 1.1.

Step 3. $\Sigma x^2 P(x) = 0+0.5+1.2 = 1.7$; $\sigma^2 = 1.7 - 1.1^2 = 0.49$; σ=0.7.

5.2 Binomial Probability Distributions

Key ideas — BINOMIAL needs:

- Fixed number of trials n.
- Each trial: success/failure (two outcomes).
- Constant success probability p.
- Independent trials.

Binomial P(X=k): $nCk \cdot p^k \cdot (1-p)^{(n-k)}$
When to use: Exactly k successes in n independent Bernoulli trials.

Binomial mean & SD: $\mu = np;\ \ \sigma = \sqrt{(np(1-p))}$
When to use: Shortcuts once you know it is binomial.

Worked Example 2: Binomial probability

Step 1. n=10 free-throws, p=0.3 make-rate. P(X=4)=10C4·(0.3)^4·(0.7)^6 = 210·0.0081·0.117649 ≈ 0.2001.

Step 2. μ=np=3; σ=√(10·0.3·0.7)=√2.1≈1.449.

Step 3. P(X≤1) = P(0)+P(1) — use calculator binomcdf for speed on tests.

TI-84 tip: TI-84: 2nd → VARS (DISTR) → binompdf(n,p,k) for exact; binomcdf(n,p,k) for P(X≤k).

5.3 Poisson Probability Distributions

- Poisson models counts in a fixed interval of time/space when events are rare/independent with mean rate λ.

Poisson P(X=k): $e^{(-\lambda)} \cdot \lambda^k / k!$

When to use: Arrivals, defects per unit, rare events; $\mu=\sigma^2=\lambda$.

Worked Example 3: Poisson

Step 1. Calls arrive at λ=3 per hour. P(X=2)=e^(−3)·9/2 ≈ 0.0498·4.5 ≈ 0.2240.

Step 2. μ=3, σ=√3≈1.732.

Step 3. If a problem gives a rate per hour but asks about 2 hours, scale λ (e.g., 3·2=6).

TI-84 tip: TI-84: poissonpdf(λ,k) and poissoncdf(λ,k).

Common mistakes

- Using binomial when p changes or trials aren't independent.
- Forgetting nCk.
- Using binompdf when the question asks ≤ or ≥ (need cdf / complement).
- Forgetting to rescale λ for a different interval.

Practice Problems

1. Can P(x)=0.4,0.4,0.3 for x=1,2,3 be a distribution?
2. μ for x=1,2,3 with P=0.2,0.5,0.3?
3. Binomial n=5,p=0.5: P(X=3)?
4. Same: μ and σ?
5. n=20,p=0.1: P(X=0)?
6. Poisson λ=4: P(X=0)?
7. Poisson λ=4: P(X≥1)?
8. Is 'number of siblings' binomial? Why/why not?
9. TI: which command for P(X≤3) binomial?
10. If λ=2.5 per day, λ for 3 days?

Answer Key — Chapter 5

1. No — probabilities sum to 1.1.
2. μ=2.1.
3. 10/32=0.3125.
4. μ=2.5; σ=√1.25≈1.118.
5. (0.9)^20≈0.1216.
6. e^(−4)≈0.0183.
7. 1−0.0183≈0.9817.
8. Usually no — n isn't fixed trials with constant p in the binomial sense.
9. binomcdf(n,p,3).
10. 7.5.

Cheat Sheet — Chapter 5

One-page review. Fold this or screenshot it before the test.

Valid distribution

- P≥0 and ΣP=1.

Mean/SD

- μ=ΣxP(x); σ=√(Σx²P−μ²).

Binomial

- n,p fixed; P(X=k)=nCk p^k (1−p)^(n−k); μ=np.

Poisson

- P(X=k)=e^(−λ)λ^k/k!; μ=λ.

TI-84

- binompdf/cdf, poissonpdf/cdf.

Chapter 6 — Normal Probability Distributions

The bell curve is the VIP of intro stats. Learn z-scores, Table A-2 / invNorm, and the Central Limit Theorem — then you can handle most of Chapters 7–9.

6.1 The Standard Normal Distribution

• Standard normal: mean 0, SD 1. Random variable Z. Areas under the curve are probabilities.

• Total area = 1. Symmetric about 0. Empirical rule: about 68% within 1 SD, 95% within 2, 99.7% within 3.

Standardize: $z = (x - \mu) / \sigma$

When to use: Converting any normal value to a standard z so you can use tables or a calculator.

Worked Example 1: Standard normal areas

Step 1. P(Z < 1.25) ≈ 0.8944 (from table/calculator).

Step 2. P(Z > −0.80) = P(Z < 0.80) ≈ 0.7881 by symmetry.

Step 3. P(−1 < Z < 1) ≈ 0.6827 ≈ 68% (empirical rule).

TI-84 tip: TI-84: normalcdf(lower,upper,0,1). For 'less than z' use lower = −1E99. invNorm(area,0,1) finds z from left-tail area.

6.2 Real Applications of Normal Distributions

Worked Example 2: IQ-style normal

Step 1. IQ ~ N(μ=100, σ=15). P(X > 115): z=(115−100)/15=1.00 → P(Z>1)≈0.1587.

Step 2. Find 90th percentile: z≈1.282 for left area 0.90 → x=100+1.282·15≈119.2.

Step 3. Always draw a sketch and shade — it prevents 'wrong tail' mistakes.

TI-84 tip: TI-84: normalcdf(115,1E99,100,15) or invNorm(0.90,100,15).

6.3 Sampling Distributions and Estimators

• A sampling distribution is the distribution of a statistic (like $\bar{x}$ or $\hat{p}$) over many random samples.

• Unbiased estimators have mean equal to the target parameter ($\bar{x}$ for μ, $\hat{p}$ for p, s^2 for σ^2 — note s itself is slightly biased).

6.4 The Central Limit Theorem

• For large n (often n≥30), $\bar{x}$ is approximately normal with mean μ and SE = σ/√n, even if the population isn't normal. If the population is normal, $\bar{x}$ is normal for any n.

CLT for means: $\bar{x} \approx N(\mu, \sigma/\sqrt{n})$

When to use: Probabilities about sample averages; use z=($\bar{x}$−μ)/(σ/√n).

Worked Example 3: CLT calculation

Step 1. Population μ=50, σ=12. Sample n=36. P($\bar{x}$ > 54).

Step 2. SE=12/√36=2. z=(54−50)/2=2.00. P(Z>2)≈0.0228.

Step 3. Notice: individual X would use σ=12; the mean uses the smaller SE=2 — averages are less variable.

6.5 Assessing Normality

• Check histogram shape, outliers, and a normal quantile plot (points should hug a straight line).

6.6 Normal as Approximation to Binomial (download-only in 14e)

• When np≥5 and n(1−p)≥5 (Triola's common check), binomial can be approximated by normal with μ=np, σ=√(npq), often with continuity correction ±0.5.

Common mistakes

• Using σ instead of σ/√n for $\bar{x}$.
• Shading the wrong tail.
• Applying CLT with tiny n when the population is clearly skewed.
• Mixing up invNorm area (must be left-tail area).

Practice Problems

1. P(Z<0)?
2. Empirical rule: % within 2 SDs?
3. N(50,4): z for x=58?
4. N(100,15): P(X<85) approx?
5. Find z with left area 0.975.
6. CLT: μ=80,σ=9,n=36. SE of $\bar{x}$?
7. Same: P($\bar{x}$<77)?
8. Why is SE smaller than σ?
9. True/false: CLT says the population becomes normal.
10. invNorm on TI-84 needs which area?

Answer Key — Chapter 6

1. 0.5.
2. About 95%.
3. z=2.
4. z=−1 → ≈0.1587.
5. ≈1.96.
6. 9/6=1.5.
7. z=(77−80)/1.5=−2 → ≈0.0228.
8. Averaging cancels noise; SD of $\bar{x}$ is σ/√n.
9. False — it says the sampling distribution of $\bar{x}$ is approx normal.
10. Left-tail cumulative area.

Cheat Sheet — Chapter 6

One-page review. Fold this or screenshot it before the test.

z

- z=(x−μ)/σ; areas via normalcdf / tables.

Empirical

- 68–95–99.7.

CLT

- $\bar{x} \sim N(\mu, \sigma/\sqrt{n})$ for large n.

TI-84

- normalcdf(low,up,μ,σ); invNorm(left area,μ,σ).

Sketch

- Always shade before you compute.

Chapter 7 — Estimating Parameters and Determining Sample Sizes

Confidence intervals turn a sample into a plausible range for a population number. Learn the pattern: point estimate ± margin of error.

7.1 Estimating a Population Proportion

CI for p: $\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$

When to use: Categorical data; need $n\hat{p} \geq 5$ and $n(1-\hat{p}) \geq 5$ (Triola-style check).

Sample size for p: $n = [z_{\alpha/2}^2 \cdot \hat{p}(1-\hat{p})] / E^2$ (use $\hat{p}=0.5$ if unknown)

When to use: Planning a survey to hit margin of error E.

Worked Example 1: Proportion CI + sample size

Step 1. 82 successes in 200 trials: $\hat{p}=0.41$. 95% CI uses z=1.96. SE=$\sqrt{(0.41 \cdot 0.59/200)} \approx 0.0348$. ME≈0.0682.

Step 2. CI ≈ (0.3418, 0.4782). Interpret: we are 95% confident the interval captures the true p.

Step 3. Want E=0.03 at 95% with no prior $\hat{p}$: $n=(1.96 \cdot 0.5/0.03)^2 \approx 1067.1 \rightarrow n=1068$.

TI-84 tip: TI-84: STAT → TESTS → 1-PropZInt.

7.2 Estimating a Population Mean

• If σ is unknown (usual case), use t with df=n−1: $\bar{x} \pm t_{\alpha/2} \cdot (s/\sqrt{n})$. Population roughly normal or n large.

CI for μ (σ unknown): $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$

When to use: Quantitative data; df = n−1.

Worked Example 2: Mean t interval

Step 1. n=25, $\bar{x}$=68.2, s=4.5, 95%. df=24, t≈2.064.

Step 2. SE=4.5/5=0.9; ME≈1.858. CI ≈ (66.34, 70.06).

Step 3. Wider s or smaller n → wider interval. Higher confidence → wider interval.

TI-84 tip: TI-84: STAT → TESTS → TInterval.

7.3 Estimating a Population Standard Deviation or Variance

• Chi-square based intervals for σ or σ^2 require a normal population. Critical values χ^2_L and χ^2_R are not symmetric.

CI for σ^2: $((n-1)s^2)/\chi^2_R < \sigma^2 < ((n-1)s^2)/\chi^2_L$

When to use: When estimating variance/SD; take square roots for σ.

7.4 Bootstrapping: Using Technology for Estimates

• Resample the sample many times with replacement, rebuild the statistic, and use the middle 95% of that bootstrap distribution as an interval — great when formulas are messy.

Common mistakes

• Saying 'the probability that μ is in this interval is 95%' (μ is fixed; the interval is random).

• Using z when s is used for a mean (use t).

• Forgetting to round sample size UP.

• Using $\hat{p}$=0.2 in sample-size formula when you truly have no estimate (0.5 is safest).

Practice Problems

1. $\hat{p}$ for 40 yes out of 100?
2. 95% z critical value?
3. n=100,$\hat{p}$=0.4: SE of $\hat{p}$?
4. Why round n up for sample size?
5. df for a t-interval with n=18?
6. Does 99% CI wider than 95% (same data)?
7. TInterval vs ZInterval: when T?
8. E=0.04,95%,$\hat{p}$ unknown: n?
9. Interpret CI in one careful sentence.
10. Bootstrap CI needs what tech idea?

Answer Key — Chapter 7

1. 0.40.
2. 1.96.
3. √(0.4·0.6/100)=√0.0024≈0.0490.
4. So the margin of error is at most E.
5. 17.
6. Yes.
7. σ unknown (use s).
8. $(1.96 \cdot 0.5/0.04)^2$=600.25→601.
9. We are __% confident the interval contains the parameter.
10. Resample with replacement many times.

Cheat Sheet — Chapter 7

One-page review. Fold this or screenshot it before the test.

Pattern

- estimate ± critical·SE.

Proportion

- $\hat{p} \pm z\sqrt{\hat{p}\hat{q}/n}$; $n=z^2\hat{p}\hat{q}/E^2$.

Mean

- $\bar{x} \pm t(s/\sqrt{n})$, df=n−1.

Tradeoffs

- More confidence or smaller E → larger n / wider CI.

TI

- 1-PropZInt, TInterval.

Chapter 8 — Hypothesis Testing

Hypothesis tests ask: if the null claim were true, how surprising is our sample? Small p-value → evidence against H_0. Stay calm, name your tails, and follow the steps.

8.1 Basics of Hypothesis Testing

Key ideas

• H_0: statement of no change/no effect, always with = (≥ or ≤ in some texts for one-sided setups, but the equality is tested).

• H_1 (H_a): what you suspect — ≠ (two-tailed), > or < (one-tailed).

• α = significance level (often 0.05). p-value = probability, assuming H_0, of a result at least as extreme as observed.

• Reject H_0 if p-value ≤ α (or if test statistic falls in the critical region).

• Type I error: reject a true H_0. Type II: fail to reject a false H_0. Power = 1 − β.

8.2 Testing a Claim About a Proportion

z for proportion: $z = (\hat{p} - p_0) / \sqrt{(p_0(1-p_0)/n)}$

When to use: Testing H_0: $p = p_0$ with adequate np_0 and $n(1-p_0)$.

Worked Example 1: Proportion test

Step 1. Claim: majority support (p>0.5). Data: 72 yes of 120. Test H_0: p=0.5 vs H_1: p>0.5 at α=0.05.

Step 2. $\hat{p}$=0.6; z=(0.6−0.5)/√(0.5·0.5/120)=0.1/√(0.002083)≈0.1/0.04564≈2.191.

Step 3. Right-tail p≈0.0142 < 0.05 → reject H_0. Evidence of majority support. (Two-sided would be ≈0.0285.)

TI-84 tip: TI-84: STAT → TESTS → 1-PropZTest.

8.3 Testing a Claim About a Mean

t for mean: $t = (\bar{x} - \mu_0) / (s/\sqrt{n})$, $df = n-1$

When to use: When σ is unknown — the usual mean test.

Worked Example 2: Mean t-test

Step 1. H_0: μ=98.6 vs H_1: μ≠98.6. n=40, $\bar{x}$=98.2, s=0.7, α=0.05.

Step 2. t=(98.2−98.6)/(0.7/√40)= (−0.4)/0.1107 ≈ −3.614, df=39.

Step 3. |t| is large; two-tailed p is tiny (≪0.05) → reject H_0. Mean appears below 98.6.

TI-84 tip: TI-84: T-Test from STAT → TESTS.

8.4 Testing a Claim About a Standard Deviation or Variance

χ^2 for variance: $\chi^2 = (n-1)s^2 / \sigma_0^2$

When to use: Normal population assumed; compare to chi-square critical values/p-values.

8.5 Resampling: Using Technology for Hypothesis Testing

• Randomization/bootstrap tests simulate the null world many times and see how rare your statistic is — same logic as a p-value, fewer formula assumptions.

Worked Example 3: Decision language

Step 1. Never say 'accept H_0.' Say 'fail to reject H_0' — you didn't prove H_0 true.

Step 2. Practical vs statistical significance: a tiny p-value with a tiny effect may not matter in real life.

Step 3. Always state hypotheses, α, test used, statistic, p-value, decision, and a sentence in context.

Common mistakes

- Putting the research claim in H_0.
- Using $\hat{p}$ in the SE for a proportion test (use p_0).
- Two-tailed vs one-tailed mix-ups.
- Saying 'accept H_0.'
- Ignoring conditions (normality, np checks).

Practice Problems

1. Write H_0/H_1 for 'mean wait < 5 min'.
2. α=0.05: reject if p=0.03?
3. $\hat{p}$=0.42,n=200,p_0=0.5: z?
4. Name a Type I error in a spam filter context.
5. df for t-test n=25?
6. True/false: p-value is P(H_0 true).
7. When use T-Test not Z-Test for a mean?
8. If fail to reject, did we prove H_0?
9. Critical z for left-tail α=0.05?
10. What does power mean?

Answer Key — Chapter 8

1. H_0: μ=5 (or μ≥5); H_1: μ<5.
2. Yes.
3. z=(0.42−0.5)/√(0.25/200)= (−0.08)/0.03536≈−2.26.
4. Flagging a good email as spam (false positive).
5. 24.
6. False — it's a probability calculated assuming H_0.
7. σ unknown.
8. No.
9. −1.645.
10. Chance of rejecting H_0 when H_1 is true.

Cheat Sheet — Chapter 8

One-page review. Fold this or screenshot it before the test.

Steps

- Hypotheses → α → check conditions → statistic → p/critical → decide → conclude in words.

Proportion

- $z=(\hat{p}-p_o)/\sqrt{(p_o q_o/n)}$.

Mean

- $t=(\bar{x}-\mu_o)/(s/\sqrt{n})$, $df=n-1$.

Decision

- $p \le \alpha \rightarrow$ reject H_o.

Wording

- Fail to reject, don't 'accept.'

Chapter 9 — Inferences from Two Samples

Two groups? First ask: independent samples or matched pairs? That choice picks your procedure.

9.1 Two Proportions

z for p_1-p_2: $z=(\hat{p}_1-\hat{p}_2)/\sqrt{(\bar{p}(1-\bar{p})(1/n_1+1/n_2))},\ \bar{p}=(x_1+x_2)/(n_1+n_2)$
When to use: Testing H_0: $p_1=p_2$. For CIs, use unpooled SE with each $\hat{p}$.

Worked Example 1: Two proportions

Step 1. Group A: 45/100, Group B: 30/100. H_0: $p_1=p_2$ vs H_1: $p_1\neq p_2$.

Step 2. $\bar{p}=75/200=0.375$; $z=(0.45-0.30)/\sqrt{(0.375\cdot0.625\cdot0.02)}=0.15/\sqrt{0.0046875}=0.15/0.06847\approx2.191$.

Step 3. Two-tailed $p\approx0.0285$ → significant at $\alpha=0.05$.

TI-84 tip: TI-84: 2-PropZTest / 2-PropZInt.

9.2 Two Means: Independent Samples

t for $\mu_1-\mu_2$: $t=(\bar{x}_1-\bar{x}_2)/\sqrt{(s_1^2/n_1+s_2^2/n_2)}$
When to use: Independent quantitative samples; df from technology (or conservative $\min(n_1,n_2)-1$).

Worked Example 2: Two independent means

Step 1. $\bar{x}_1=82, s_1=6, n_1=20$; $\bar{x}_2=78, s_2=5.5, n_2=22$. H_0: $\mu_1=\mu_2$.

Step 2. $SE=\sqrt{(36/20+30.25/22)}=\sqrt{(1.8+1.375)}=\sqrt{3.175}\approx1.782$; $t\approx4/1.782\approx2.245$.

Step 3. Positive t: group 1 mean looks higher; compare p to α with calculator df.

TI-84 tip: TI-84: 2-SampTTest / 2-SampTInt. Prefer 'not pooled' unless told variances are equal.

9.3 Matched Pairs

• Same subjects twice, or natural pairs. Analyze differences d_i = after−before (or matched gap). Then one-sample t on the differences.

Paired t: $t=\bar{d}/(s_d/\sqrt{n}),\ df=n-1$
When to use: Before/after or matched designs.

Worked Example 3: Paired differences

Step 1. Differences: 2, −1, 3, 4, 1, 2, 0, 3. $n=8$, $\bar{d}=1.75$, $s_d\approx1.669$.

Step 2. $t=1.75/(1.669/\sqrt{8})=1.75/0.590\approx2.966$, $df=7$.

Step 3. Evidence of a positive mean change if testing H_0: $\mu_d=0$ vs H_1: $\mu_d>0$ (p small).

TI-84 tip: Enter differences in a list, then T-Test / TInterval.

9.4 Two Variances or Standard Deviations

F-test: $F=s_1^2/s_2^2$ (put larger s^2 on top for a two-tailed convenience approach)
When to use: Comparing variability; sensitive to non-normality.

9.5 Resampling: Using Technology for Inferences

• Randomization tests shuffle labels to simulate the null of 'no difference.' Bootstrap CIs resample within groups.

Common mistakes

- Using two-sample t on paired data (or vice versa).
- Pooling proportions incorrectly for a CI.
- Pooling variances by default.
- Forgetting which group is which in subtraction.

Practice Problems

1. Independent or paired: twin study, one twin each diet?
2. Independent or paired: random men vs random women heights?
3. Pooled $\bar{p}$ for 10/50 and 20/60?
4. Paired data: what list do you analyze?
5. 2-PropZTest tests what H_0?
6. True/false: matched pairs reduce subject-to-subject noise.
7. For $\mu_1-\mu_2$ CI, is pooled always required?
8. $F= s_1^2/s_2^2$ compares what?
9. If differences are all ~0, t is near?
10. Name the TI test for paired means.

Answer Key — Chapter 9

1. Paired.
2. Independent.
3. 30/110≈0.2727.
4. The differences.
5. $p_1=p_2$.
6. True.
7. No.
8. Variances (SDs).
9. 0.
10. T-Test on the difference list (or Data with L1−L2 setup).

Cheat Sheet — Chapter 9

One-page review. Fold this or screenshot it before the test.

First question

- Independent or paired?

Two proportions

- z with pooled $\bar{p}$ for tests.

Two means

- 2-SampTTest; usually unpooled.

Paired

- t on differences.

Variances

- $F = s_1^2/s_2^2$.

Chapter 10 — Correlation and Regression

Scatterplots tell the story; r measures linear strength; the regression line predicts. Still: correlation ≠ causation!

10.1 Correlation

Linear correlation r: $r = \Sigma[(x-\bar{x})(y-\bar{y})] / \sqrt{[\Sigma(x-\bar{x})^2\ \Sigma(y-\bar{y})^2]}$

When to use: Measuring strength/direction of linear association; $-1 \le r \le 1$.

- $|r|$ near 1: strong linear. Near 0: weak linear (there could still be a curve!). Test H_0: $\rho=0$ with a t based on r.

Worked Example 1: Compute r (small set)

Step 1. x: 2,3,5,7,9 and y: 4,5,8,10,13.

Step 2. Means $\bar{x}=5.2$, $\bar{y}=8$. $\Sigma(x-\bar{x})(y-\bar{y})=52.0$; $\sqrt{[\Sigma(x-\bar{x})^2\ \Sigma(y-\bar{y})^2]}=\sqrt{[32.8\cdot52.0]}=\sqrt{1705.6}\approx41.299$.

Step 3. $r \approx 0.998$ (very strong positive). Always graph — a single influential point can inflate r.

TI-84 tip: TI-84: turn on DiagnosticOn (Catalog), then STAT → CALC → LinReg(ax+b) L1,L2.

10.2 Regression

Least-squares line: $\hat{y} = b_0 + b_1 x,\ b_1 = r\cdot(s_y/s_x),\ b_0 = \bar{y} - b_1\bar{x}$

When to use: Predicting y from x when the linear model is appropriate.

Worked Example 2: Regression line + prediction

Step 1. Using the same points, slope $b_1\approx1.280$, intercept $b_0\approx1.341$, so $\hat{y}\approx1.341+1.280x$.

Step 2. Predict at $x=6$: $\hat{y}\approx1.341+7.68\approx9.02$.

Step 3. Don't extrapolate far beyond the x-range (here about 2 to 9).

10.3 Prediction Intervals and Variation

- r^2 (coefficient of determination) = fraction of y-variation explained by the linear relation with x. Prediction intervals for a single future y are wider than confidence intervals for the mean response.

10.4 Multiple Regression

- More than one predictor: $\hat{y} = b_0 + b_1x_1 + b_2x_2 + \ldots$ Technology required. Check adjusted R^2 and whether predictors are significant.

10.5 Nonlinear Regression

- Curves (quadratic, exponential, etc.) can fit better than a straight line — use residual plots and scatter shape to decide.

Worked Example 3: Interpret r and r^2

Step 1. If $r=-0.8$, $r^2=0.64$: strong negative linear association; 64% of y-variation is explained by the linear model with x.

Step 2. A residual plot with a clear curve means 'linear' is the wrong form even if $|r|$ looks okay.

Step 3. Outliers and influential points can swing r and the slope — always look at the graph.

Common mistakes

- Causation claims from r alone.
- Extrapolating wildly.
- Ignoring a curved pattern because |r| isn't zero.
- Forgetting DiagnosticOn so r doesn't show on older OS workflows.

Practice Problems

1. Range of r?
2. r=0 means no relationship at all?
3. r^2=0.49 means?
4. Slope of $\hat{y}=5-2x$ means?
5. Extrapolate: good idea?
6. TI command for line?
7. Multiple regression needs?
8. Residual plot purpose?
9. If y increases as x increases, r sign?
10. Correlation unitless?

Answer Key — Chapter 10

1. −1 to 1.
2. No — only no linear relationship.
3. 49% of y variation explained linearly by x.
4. Each +1 in x predicts −2 in y.
5. Usually no.
6. LinReg(ax+b).
7. Technology / matrix methods.
8. Check linearity and constant spread.
9. Positive.
10. Yes.

Cheat Sheet — Chapter 10

One-page review. Fold this or screenshot it before the test.

r

- −1…1 linear strength/direction.

Line

- $\hat{y}=b_0+b_1x$; $b_1=r\ s_y/s_x$.

r^2

- Explained variation fraction.

Warnings

- No causation; no wild extrapolation; check residual plots.

TI

- DiagnosticOn + LinReg.

Chapter 11 — Goodness-of-Fit and Contingency Tables

Chi-square tests compare observed counts to expected counts. One variable → goodness-of-fit. Two categorical variables → contingency table (independence/homogeneity).

11.1 Goodness-of-Fit

χ^2 GOF: $\chi^2 = \Sigma[(O-E)^2/E]$, $df = k-1$

When to use: Testing whether counts follow a claimed distribution (uniform or specified).

• Need expected counts usually ≥5. Right-tailed test (big χ^2 = bad fit).

Worked Example 1: Uniform GOF

Step 1. n=80 across 4 categories, H_0: equal proportions. E=20 each. O=25,18,22,15.

Step 2. $\chi^2=(5)^2/20+(-2)^2/20+(2)^2/20+(-5)^2/20=25/20+4/20+4/20+25/20=2.9$.

Step 3. df=3. $\chi^2=2.9$ is small → fail to reject (data look compatible with equal shares).

TI-84 tip: TI-84: enter O in L1, E in L2 → STAT → TESTS → χ^2GOF-Test.

11.2 Contingency Tables

Expected cell: $E = (\text{row total})(\text{column total})/n$

When to use: Building the comparison table for independence tests.

χ^2 association: $\chi^2=\Sigma[(O-E)^2/E]$, $df=(r-1)(c-1)$

When to use: H_0: variables independent (or populations homogeneous).

Worked Example 2: 2×2 contingency

Step 1. Table: [[30,20],[10,40]]; row totals 50,50; column totals 40,60; n=100.

Step 2. E11=50·40/100=20; E12=30; E21=20; E22=30.

Step 3. $\chi^2=(30-20)^2/20+(20-30)^2/30+(10-20)^2/20+(40-30)^2/30=5+3.333+5+3.333=16.667$, df=1 → strong evidence of association.

TI-84 tip: TI-84: enter matrix of observed → χ^2-Test.

Worked Example 3: Independence vs GOF

Step 1. GOF: one categorical variable vs a theorized breakdown.

Step 2. Contingency: two categorical variables — are they related?

Step 3. Both use $(O-E)^2/E$ and are right-tailed. Read the story before picking the test.

Common mistakes

• Using χ^2 on averages instead of counts.
• df wrong (k−1 vs (r−1)(c−1)).
• Expected counts too small.
• Two-tailed thinking — χ^2 GOF/independence are right-tailed.

Practice Problems

1. GOF df for 6 categories?

2. E if uniform n=90, k=5?

3. Contingency df for 3×4 table?
4. E formula?
5. χ^2 large means?
6. Need raw counts or percents?
7. TI test for matrix table?
8. H_0 in independence test?
9. If O=E everywhere, χ^2=?
10. Minimum E rule of thumb?

Answer Key — Chapter 11

1. 5.
2. 18.
3. 6.
4. row tot · col tot / n.
5. Poor fit / evidence of association.
6. Counts.
7. χ^2-Test.
8. Variables independent.
9. 0.
10. Often E≥5.

Cheat Sheet — Chapter 11

One-page review. Fold this or screenshot it before the test.

GOF

- $\chi^2=\Sigma(O-E)^2/E$; df=k−1.

Two-way

- E=row·col/n; df=(r−1)(c−1).

Tail

- Right-tailed.

TI

- χ^2GOF-Test; χ^2-Test with matrix.

Chapter 12 — Analysis of Variance

ANOVA compares means across 3+ groups without running a forest of t-tests. The key idea: is the variation between groups large compared to variation within groups?

12.1 One-Way ANOVA

Key ideas

• H_0: $\mu_1=\mu_2=\ldots=\mu_k$. H_1: at least one mean differs.

• F = MS(between) / MS(within). Large F → group means differ more than random noise explains.

• Assumptions: independent random samples, roughly normal populations, similar variances (rule of thumb: largest s not much more than twice smallest).

• If you reject H_0, follow up with pairwise comparisons (technology) to see which groups differ.

ANOVA F: $F = MS_B / MS_W,\ \ MS_B = SS_B/(k-1),\ \ MS_W = SS_W/(n-k)$

When to use: Testing equality of several means (one factor).

Worked Example 1: Tiny ANOVA by hand

Step 1. Groups: A 10,12,11,13; B 15,14,16,15; C 20,19,21,18. k=3, n=12.

Step 2. Grand mean=15. SS_B=128.667, SS_W=12.0, SS_T=140.667.

Step 3. MS_B=128.667/2=64.333; MS_W=12/9=1.333; F≈48.25 → reject H_0 (means clearly differ).

TI-84 tip: TI-84: enter groups in L1–L3 → STAT → TESTS → ANOVA(L1,L2,L3).

Worked Example 2: Reading an ANOVA table

Step 1. Look at df, SS, MS, F, and p-value.

Step 2. If p ≤ α, conclude at least one mean differs — but ANOVA alone doesn't say which.

Step 3. If variances look wildly unequal or shapes are very skewed with small n, be cautious.

12.2 Two-Way ANOVA

• Two factors (e.g., teaching method and time of day). You can test each main effect and the interaction.

• Interaction means the effect of Factor A depends on the level of Factor B — always look at an interaction plot.

Worked Example 3: Interaction intuition

Step 1. Coffee may help morning exam scores a lot but evening scores barely — that's an interaction between drink and time.

Step 2. If lines on an interaction plot are parallel, interaction is weak/absent.

Step 3. Two-way ANOVA is almost always done with software/calculator, not by hand.

Common mistakes

• Claiming ANOVA identifies which pair differs without a follow-up.

• Running many t-tests instead (inflates Type I error).

• Ignoring unequal variance / tiny samples.

• Mixing up MS_B and MS_W in F.

Practice Problems

1. H_0 for one-way ANOVA?
2. F = ratio of what?
3. df between for 4 groups?
4. df within if n=40,k=4?
5. Large F suggests?
6. Why not many t-tests?
7. Two-way ANOVA can test?
8. Interaction means?
9. TI ANOVA input?
10. After rejecting, next step?

Answer Key — Chapter 12

1. All population means equal.
2. MS between / MS within.
3. 3.
4. 36.
5. At least one mean differs.
6. Type I error multiplies.
7. Main effects + interaction.
8. Effect of one factor depends on the other.
9. ANOVA(L1,L2,…).
10. Multiple comparisons / pairwise tests.

Cheat Sheet — Chapter 12

One-page review. Fold this or screenshot it before the test.

One-way

- H_0: all μ equal; F=MS_B/MS_W.

df

- k−1 and n−k.

Follow-up

- Reject → pairwise comparisons.

Two-way

- Main effects + interaction.

TI

- ANOVA(lists).

Chapter 13 — Nonparametric Tests

Nonparametric methods use ranks or signs. They're handy when data are ordinal, skewed, or assumptions for t/ANOVA fail. Tradeoff: often less power when normal models truly fit.

13.1 Basics of Nonparametric Tests

• Fewer assumptions about population shape. Often require independent samples / random sampling still. Results can be based on medians or distributions rather than means.

13.2 Sign Test

• Counts pluses/minuses vs a hypothesized median, or signs of paired differences. Binomial (or normal approx) under H_0.

13.3 Wilcoxon Signed Ranks Test for Matched Pairs

• Like a paired t, but uses ranks of absolute differences with signs restored. More efficient than the plain sign test because it uses magnitude order.

13.4 Wilcoxon Rank-Sum Test for Two Independent Samples

• Rank all values together, sum ranks for one sample (Mann–Whitney equivalent). Alternative to two-sample t.

13.5 Kruskal-Wallis Test for Three or More Samples

Kruskal-Wallis idea: H statistic from rank sums across groups (χ^2 approx)

When to use: Nonparametric cousin of one-way ANOVA.

13.6 Rank Correlation

Spearman r_s: Correlate the ranks of x with the ranks of y

When to use: Monotonic association; resistant to some outliers / nonlinear monotone patterns.

13.7 Runs Test for Randomness

• A run is a streak of similar outcomes (e.g., MMMFFFF). Too few or too many runs suggests non-random order.

Worked Example 1: Pick the nonparametric tool

Step 1. Paired ordinal satisfaction before/after → sign test or Wilcoxon signed ranks.

Step 2. Two independent skewed income samples → Wilcoxon rank-sum.

Step 3. Three teaching methods with ordinal scores → Kruskal-Wallis.

Worked Example 2: Sign test sketch

Step 1. Hypothesized median commute = 30 min. Differences vs 30: seven +, two − (ignore zeros).

Step 2. Under H_0, P(+) = 0.5. Seeing 7 of 9 positives is somewhat unusual — compute binomial p.

Step 3. If $p \le \alpha$, evidence median ≠ 30 (or >30 for one-sided).

Worked Example 3: When NOT to go nonparametric

Step 1. If data are roughly normal, n is decent, and you care about means, a t-test/ANOVA often has more power.

Step 2. Nonparametric ≠ 'no assumptions at all.' Independence still matters.

Step 3. Always match the design: paired vs independent still applies!

Common mistakes

- Using Kruskal-Wallis on paired repeated measures without a paired method.
- Throwing away magnitude when Wilcoxon signed ranks is available (using only signs).
- Forgetting to drop zeros in a sign test.
- Treating Spearman like Pearson for linear-only questions without looking at the scatter.

Practice Problems

1. Nonparametric methods often use what instead of raw values?
2. Sign test focuses on?
3. Wilcoxon signed ranks is for?
4. Rank-sum is alternative to?
5. Kruskal-Wallis alternative to?
6. Spearman uses?
7. Runs test checks?
8. Tradeoff vs parametric?
9. Ignore zeros in sign test?
10. True/false: nonparametric means assumption-free.

Answer Key — Chapter 13

1. Ranks/signs.
2. Signs vs median or paired change.
3. Matched pairs.
4. Two-sample t.
5. One-way ANOVA.
6. Ranks of the two variables.
7. Randomness of a sequence.
8. Often less power when parametric assumptions hold.
9. Yes.
10. False.

Cheat Sheet — Chapter 13

One-page review. Fold this or screenshot it before the test.

Why

- Skew, ordinal data, assumption worries.

Paired

- Sign test; Wilcoxon signed ranks.

Two groups

- Wilcoxon rank-sum.

3+ groups

- Kruskal-Wallis.

Extra

- Spearman r_s; runs test for randomness.

Chapter 14 — Statistics Process Control

Note: Pearson's 14e title is "Statistics Process Control." SPC watches a process over time so you can spot when it goes 'out of control' — not just describe a static sample.

14.1 Control Charts for Variation and Mean

Key ideas

• An R-chart monitors process variability (ranges of small samples over time). An $\bar{x}$-chart monitors the process mean.

• Centerline = process average (from data while 'in control'). UCL/LCL sit roughly near ±3 SE (formulas use table constants A_2, D_3, D_4 with mean range $\bar{R}$).

• Out-of-control signals: a point beyond a control limit; runs of points on one side; clear trends/cycles (rules vary by text).

• Important: a process can be in control statistically and still miss customer specs — control ≠ capability.

$\bar{x}$-chart limits (common form): $CL = \bar{\bar{x}};\ UCL = \bar{\bar{x}} + A_2\bar{R};\ LCL = \bar{\bar{x}} - A_2\bar{R}$

When to use: Monitoring mean of periodic samples; use with an R-chart.

R-chart limits: $CL = \bar{R};\ UCL = D_4\bar{R};\ LCL = D_3\bar{R}$

When to use: Monitoring within-sample ranges over time.

Worked Example 1: Reading charts

Step 1. If the R-chart is out of control, fix variability first — the $\bar{x}$-chart limits depend on $\bar{R}$.

Step 2. A single point above UCL on $\bar{x}$: investigate special causes (machine change, new operator, bad batch).

Step 3. Eight points in a row above the centerline can also be a warning even if none cross UCL.

14.2 Control Charts for Attributes

• p-chart: monitor defective proportion in samples of size n. Centerline $\bar{p}$; limits use $\sqrt{(\bar{p}(1-\bar{p})/n)}$.

• Related charts: np-chart (counts of defectives), c-chart / u-chart for defect counts.

p-chart limits: $CL = \bar{p};\ UCL/LCL = \bar{p} \pm 3\sqrt{(\bar{p}(1-\bar{p})/n)}$

When to use: Attribute data: fraction defective each period.

Worked Example 2: p-chart intuition

Step 1. Daily samples of 100 parts; average defective rate $\bar{p}=0.04$.

Step 2. SE=$\sqrt{(0.04 \cdot 0.96/100)}=\sqrt{0.000384}\approx 0.0196$; $3 \cdot SE \approx 0.0588$.

Step 3. UCL≈0.0988, LCL≈0 (can't go negative). A day with 12 defectives (0.12) would flag.

Worked Example 3: Control vs capability

Step 1. In control: stable, predictable variation from common causes only.

Step 2. Capable: output meets specification limits (customer requirements).

Step 3. You can be stably making the wrong size — stable but not capable!

Common mistakes

• Using control limits as if they were product specification limits.

• Reading $\bar{x}$-chart before checking the R-chart.

- Treating every wiggle as a crisis (common-cause noise exists).
- Forgetting sample size n affects p-chart width.

Practice Problems

1. What does an R-chart monitor?
2. What does an $\bar{x}$-chart monitor?
3. Typical control limits use about how many SEs?
4. p-chart data type?
5. If R-chart is wild, trust $\bar{x}$ limits?
6. Control = capability? True/false.
7. UCL below a point means?
8. Attribute vs variable data?
9. Centerline of p-chart?
10. Why LCL of a p-chart might be set to 0?

Answer Key — Chapter 14

1. Process variation (ranges).
2. Process mean.
3. 3.
4. Yes/no defectives → proportions.
5. No — stabilize variation first.
6. False.
7. Out-of-control signal (investigate).
8. Attribute: categories/counts; variable: measurements.
9. $\bar{p}$.
10. Because a negative proportion is impossible.

Cheat Sheet — Chapter 14

One-page review. Fold this or screenshot it before the test.

Goal

- Detect special causes over time.

Variable data

- R-chart + $\bar{x}$-chart (check R first).

Attribute data

- p-chart: $\bar{p} \pm 3\sqrt{(\bar{p}\bar{q}/n)}$.

Signals

- Beyond limits; long runs; trends.

Reminder

- In control ≠ meets specs.

Bonus Round: Final Review

Last call for the course ideas. This round is about choosing the right tool, reading results in context, and not getting fooled by flashy numbers. (Note: Pearson's public 14e TOC ends at Chapter 14; treat this as your cumulative / holistic review level.)

15.1 Choosing the Right Procedure

Key ideas

• Ask: What is the response? Categorical or quantitative? One sample, two samples, or many? Independent or paired?

• Proportions → z methods (one or two samples). Means → t methods. Counts vs theory → χ^2. Several means → ANOVA. Association of two quantitative variables → correlation/regression.

• If assumptions fail (heavy skew, ordinal ranks), remember the nonparametric cousins from Chapter 13.

• SPC charts answer a different question: is a process stable over time?

Worked Example 1: Pick the tool

Step 1. Comparing average tip amounts for Friday vs Saturday independent shifts → two-sample t (independent means).

Step 2. Same bartender's tip % before vs after a new upselling script → paired t on differences.

Step 3. Are drink-type orders (beer/wine/cocktail) matching an expected 50/30/20 mix tonight? → χ^2 goodness-of-fit.

15.2 Interpreting Results Like a Human

Key ideas

• Statistical significance ≠ practical importance. A tiny tip difference can have a tiny p-value with huge n.

• Confidence intervals answer 'what's a plausible range?' Hypothesis tests answer 'is this compatible with a claim?'

• Always state the population you can actually generalize to (who was sampled?).

• Association is not causation — especially in observational bar-shift data full of confounders (weather, events, staffing).

Worked Example 2: Read the fine print

Step 1. A 95% CI for mean Saturday sales is ($2,140, $2,380). Do not say 'P(μ is in here)=0.95' for this fixed interval.

Step 2. Say: we're 95% confident the method captured the true mean Saturday sales.

Step 3. If an ad claims 'our training causes 20% higher tips' from a voluntary Instagram poll, that's a red flag sample + causation leap.

15.3 Connecting the Big Threads

Key ideas

• Probability → sampling distributions → CLT → intervals & tests. That pipeline is the backbone of inference.

• Every test has conditions. Sketch them before you compute.

• Graphs first (Chapter 2–3), numbers second. Outliers and skew change the story.

• When in doubt: name the parameter, name the design, name the procedure, check conditions, compute, conclude in context.

Inference pattern: $\text{estimate} \pm \text{critical} \cdot \text{SE}$ or $(\text{estimate} - \text{null}) / \text{SE}$

When to use: The same skeleton under almost every z/t interval or test.

Worked Example 3: Mini case

Step 1. Claim: more than 60% of guests order an appetizer. Sample: 78 of 120 random tickets include one.

Step 2. $\hat{p}=78/120=0.65$. Test H_0: $p=0.60$ vs H_1: $p>0.60$.
$z=(0.65-0.60)/\sqrt{(0.60\cdot0.40/120)}=0.05/\sqrt{0.002}=0.05/0.04472\approx1.118$.

Step 3. One-sided $p\approx0.132 > 0.05$ → fail to reject. Suggestive bump, not strong evidence at $\alpha=0.05$.

TI-84 tip: TI-84: 1-PropZTest with $p_0=0.6$, x=78, n=120, $>p_0$.

Common mistakes

• Picking two-sample t for paired data (or the reverse).

• Causation language from observational association.

• Skipping conditions because the calculator gave a number.

• Treating p-value as 'probability the null is true.'

Practice Problems

1. Guests rate service 1–5 stars. Comparing three music playlists' mean ratings (independent nights). Which procedure family?

2. True/false: A significant result always matters for business decisions.

3. Paired or independent: each guest's wait time with old vs new seating app on the same night shift pairs?

4. Name one reason a convenience sample of regulars may bias tip estimates.

5. Pipeline fill-in: data → ___ → sampling distribution → inference.

6. If r=0.9 between hours open and sales, can we say longer hours cause more sales?

7. χ^2 or regression: relating two categorical menu-section choices?

8. What does 'fail to reject H_0' mean in plain words?

9. Give the SE for $\hat{p}$ when n=100 and $p_0=0.5$ under H_0.

10. Name two graphs you'd check before trusting a mean for skewed check totals.

Answer Key — Chapter 15

1. One-way ANOVA (or Kruskal-Wallis if ordinal/skewed assumptions worry you).

2. False — check effect size / practical impact.

3. Paired (matched on guest/ticket context as described).

4. Regulars may tip differently than tourists; not representative.

5. Probability / randomness models (and statistics like $\bar{x}$ or $\hat{p}$).

6. No — association ≠ causation; confounding likely.

7. χ^2 contingency / test of association.

8. Data aren't unusual enough (under H_0) to discard the claim at α.

9. $\sqrt{(0.5\cdot0.5/100)}=0.05$.

10. Histogram/boxplot (and maybe a time plot for shifts).

Cheat Sheet — Chapter 15

One-page review. Fold this or screenshot it before the test.

Choose

- Categorical vs quantitative; one/two/many; paired vs independent.

Backbone

- Probability → sampling distributions → CLT → CI/tests.

Pattern

- estimate ± critical·SE or (est−null)/SE.

Wisdom

- Significance ≠ importance; association ≠ causation; conditions matter.

Closeout

- Conclude in context for the population you actually sampled.